NASA TECHNICAL NOTE

NASA TN D-3170

c.1

NASA TN D-3170

POSTLAUNCH STRUCTURAL ANALYSIS OF ECHO II SATELLITE

by Hossein Bahiman

Goddard Space Flight Center
Greenbelt, Md.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1966

POSTLAUNCH STRUCTURAL ANALYSIS OF ECHO II SATELLITE

By Hossein Bahiman

Goddard Space Flight Center
Greenbelt, Md.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – Price $1.00

ABSTRACT

Telemetered data from the Echo II (1964 04A) satellite revealed an unexpectedly high spin rate. An analysis was performed to estimate the effects of the resulting centrifugal forces. Calculations were made for the case of the axis orientation that would give the greatest distortion, namely, spin about the polar axis. It was discovered from these calculations that deviation from spherical shape was greatest at the beacons. The maximum deviation was estimated at about 3 inches with local wrinkling of a fraction of an inch in depth. Wrinkling of other areas of the satellite surface was postulated as a result of a shifting spin axis during the initial orbits.

CONTENTS

POSTLAUNCH STRUCTURAL ANALYSIS OF ECHO II SATELLITE

by
Hossein Bahiman
Goddard Space Flight Center

INTRODUCTION

Prior to launch of the Echo II (1964 04A) satellite, extensive materials tests were conducted on the satellite skin and a theoretical analysis was performed to predict the degree of smoothness of the satellite's skin surface as a function of pressure. The analysis was primarily based on a nonrotating sphere and took into consideration buckling forces such as solar radiation pressure and aerodynamic pressure. The only spin condition that was considered was a gravity-gradient-induced rotation rate of no more than one revolution per orbit. The centrifugal effects of such a rotation rate were found to be negligible.

Telemetered data received during inflation of the satellite in orbit revealed that the maximum internal pressure was about 220 microns of mercury. This corresponds to a skin stress of approximately 4800 psi. It appears from results of prelaunch tests and the analysis that such a skin stress would be sufficient to smooth all the wrinkles and folds and also to eliminate almost all of the initial deviations of the shell from a true sphere. However, it has been determined that the satellite is spinning with a period of approximately 100 seconds and so may not be a true sphere. This study presents an analysis of the effect of the centrifugal forces on the surface condition of the satellite.

MATERIAL AND CONSTRUCTION OF SATELLITE

A brief review of the material and construction of the satellite is in order before the analysis is presented. The basic material of the Echo II spherical shell is a three-layer laminate composed of a core sheet of 0.35-mil Mylar bonded between two layers of 0.18-mil aluminum foil (alloy 1080-0). Reinforced material is made by bonding a 0.50-mil Mylar sheet to one side of the basic material.

The sphere contains 106 lune-shaped gores; those numbered 1, 53, 54, and 106 are made of reinforced material, while the remaining gores are of basic material (Figure 1). Adjacent gores are joined by a 1-inch-wide thermosetting adhesive-coated tape of the same basic material, centered on the line of the butted gore edges. At each end the gores converge to a 54-inch-diameter pole cap of reinforced material. (The pole cap material was made by bonding a 1-mil Mylar sheet to the basic material.)

Two beacons, each surrounded by four solar cell panels (Figure 2), are mounted midway between the pole cap on reinforced gore pairs 53-54 and 1-106. Beneath each beacon-solar-cell array, the reinforced material is further reinforced by several layers of laminate of varying diameters so that the effective diameter of the heavily reinforced area is about 52 inches.

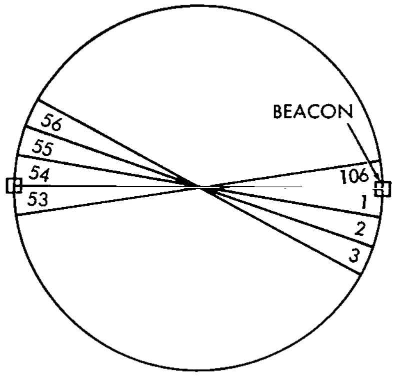

Figure 1—Echo II beacon locations.

THEORETICAL ANALYSIS

It appears from the results of tests on 30-inch spheres of Echo II material at Langley Research Center that the buckling pressure after prestressing at 3000 psi is about 15 percent of the calculated pressure P_c as given by

$$P_c = \left(\frac{2}{r}\right)^2 (DK)^{\frac{1}{2}} , \tag{1}$$

in which r is the radius, D is the bending stiffness, and K is the extensional stiffness. The calculated buckling pressure for Echo II is 6×10^{-6} psi. It could be hypothesized that the ratio of

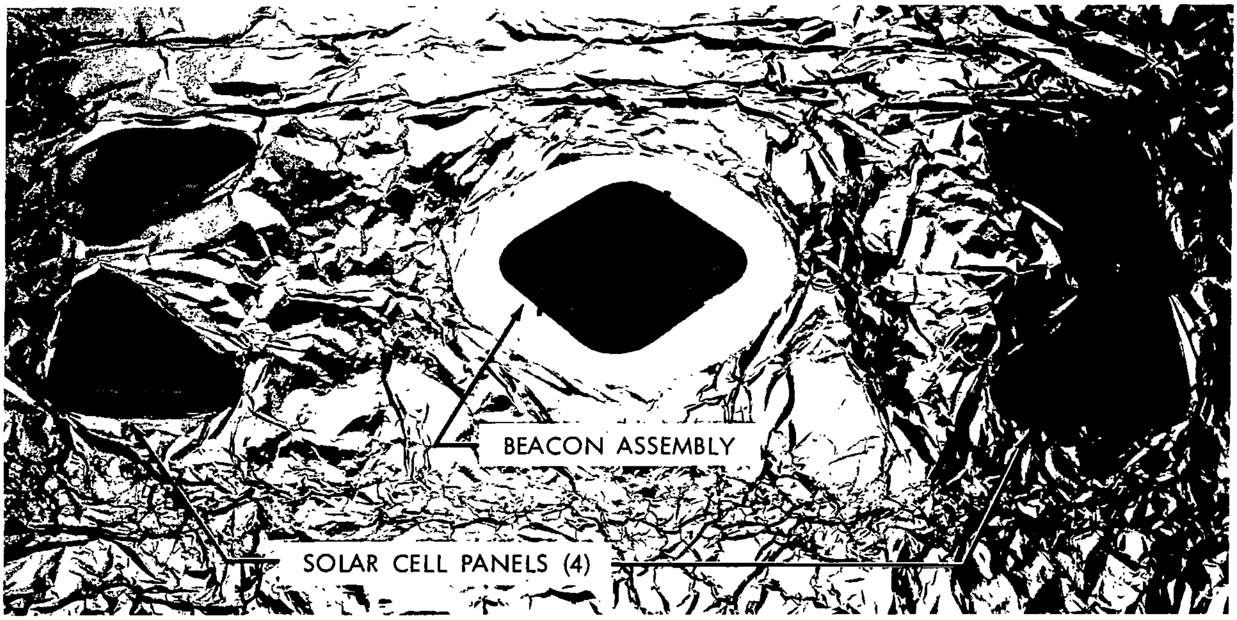

Figure 2—Beacon and solar cells mounted on sphere.

the calculated to theoretical buckling pressure of Echo II is the same as the corresponding ratio for a 30-inch-diameter sphere. On this basis a real buckling pressure of 9×10^{-7} psi will be established for Echo II. This buckling pressure corresponds to $N_{cr} = -0.00036$ lb/in.

If the effects of the beacons and the extra reinforcement at the polar regions are omitted, the stresses due to the centrifugal forces acting on the spinning satellite can be calculated as follows:

$$dF = \rho r^3 \omega^2 \sin^2 \phi \, d\phi \, d\theta \tag{2}$$

in which dF is the element of the centrifugal force, ρ is mass per unit area, r is the radius, ω is the angular velocity, ϕ is the angle between the position vector and the spin axis, and θ is the angle between the meridian through a point and an arbitrary reference meridian. It follows that

$$\left.\begin{aligned}
\rho &= \frac{563}{32.2 \times 4\pi \, (67.5)^2} = 0.000306 \, , \\
N_\phi &= 0 \, , \\
N_{\phi\theta} &= 0 \, , \\
N_\theta &= \rho \omega^2 r^2 \sin^2 \phi \, , \\
N_{\theta_{max}} &= \rho \omega^2 r^2 = 0.000306 \, (67.5)^2 \left(\frac{2\pi}{100}\right)^2 = 55 \times 10^{-4} \text{ lb/ft} \, ,
\end{aligned}\right\} \tag{3}$$

where N_ϕ, N_θ, and $N_{\phi\theta}$ are normal and shear membrane forces per unit width as shown in Figure 3.

The differential equation of displacement is as follows:

$$\frac{dV}{d\phi} - V \cot \phi = \frac{1 + \mu}{Et} \, r \left(N_\phi - N_\theta\right) = \frac{r}{K(1 - \mu)} \left(N_\phi - N_\theta\right) \tag{4}$$

where $K = Et/1 - \mu^2$ and V is meridional displacement. Solution of this equation is given by

$$V = \sin \phi \left[\frac{1}{K(1 - \mu)} \rho \omega^2 r^3 \cos \phi + C\right] \tag{5}$$

In order to find the constant of integration C apply the boundary condition, namely, $V = 0$ at $\phi = \pi/2$. Hence, $C = 0$, and as a result

$$V = \frac{0.000306 \, (0.0628)^2 \, (67.5)^3 \sin \phi \cos \phi}{800 \times 0.7} = 0.000661 \sin \phi \cos \phi \text{ ft} \tag{6}$$

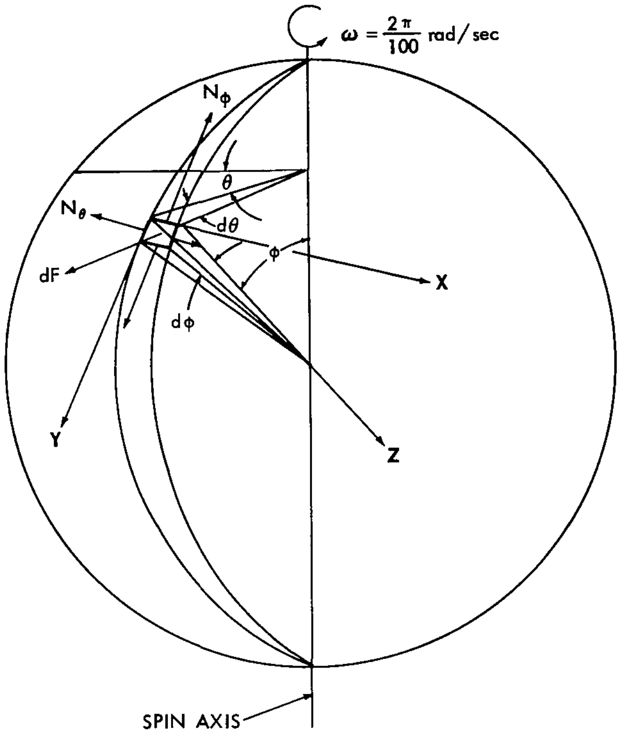

Figure 3—Forces on element of satellite shell.

and

$$\epsilon_\theta = \frac{N_\theta}{k(1 - \mu^2)}$$

$$= \frac{55 \times 10^{-4} \sin^2 \phi}{800\ (0.91)}$$

$$= 0.00000755 \sin^2 \phi \qquad (7)$$

where ϵ_θ is the strain in the direction normal to the spin axis.

The radial displacement W is given by

$$W = -r\,\epsilon_\theta + V\cot\phi$$

$$= (-0.000510 \sin^2 \phi + 0.000661 \cos^2 \phi), \qquad (8)$$

$$W|_{\text{pole}} = 0.00066 \text{ ft, inward,}$$

and (9)

$$W|_{\text{equator}} = 0.00051 \text{ ft, outward.}$$

The above displacements are quite small. Surface irregularities caused by spinning can be calculated in a simple way. To get the maximum radial displacement, let it be assumed that the spin axis passes through the pole caps and is thus normal to the beacon axis. As mentioned before, a number of concentric patches, with different average diameters, reinforce those regions of the satellite which are directly under or in the vicinity of the beacons. The extensional stiffness of the 52-inch-diameter patch under each beacon is about 23 times greater than that of the basic material of the satellite. It is assumed, here, that this region does not buckle under the existing spin.

The centrifugal force -R acting on the beacon may be calculated as follows:

$$R = -F = mr\omega^2 = \left(\frac{7.5}{32.2}\right) (67.5) \left(\frac{2\pi}{100}\right)^2 = 0.062 \text{ lb} \qquad (10)$$

where 7.5/32.2 is the total of the mass of the beacon and the equivalent mass of the reinforcement patch concentrated at the center of the beacon. Neglecting the centrifugal body force on the membrane causes the radial body force Z to become zero. Hence and

$$\left.\begin{aligned} N_{\phi'} &= \frac{0.062}{810(2\pi)\sin^2\phi'} , \qquad \sin\phi' \geqq \frac{26}{810} , \\ N_{\theta'} &= rZ - N_{\phi'} , \\ N_{\theta'} &= -N_{\phi'} = \frac{-0.0000122}{\sin^2\phi'} \end{aligned}\right\} \tag{11}$$

Consideration of the above stress $N_{\theta'}$ in conjunction with N_ϕ and N_θ given by Equations 3 leads to

$$N_\phi + N_{\theta'} = \frac{-0.0000122}{\sin^2\phi'} = -0.00036 \tag{12}$$

which gives $\phi' = 10.6°$ in the equatorial plane. The total normal force per unit length in the meridional plane is given by

$$N_{\theta'} + N_\theta = \frac{-0.0000122}{\sin^2\phi'} + \frac{0.0055}{12}\cos^2\phi' = -0.00036 . \tag{13}$$

For an approximate solution, the substitution of $\cos^2\phi' = 0.97$ in Equation 13 results in

$$\frac{-0.0000122}{\sin^2\phi'} = -0.00080 \tag{14}$$

which gives $\phi' = 7.1°$. The two angles 10.6° and 7.1° are half the subtending angle of the conoidal surface of buckling in the equatorial and meridional plane, respectively. Hence, the buckled conoidal area has a roughly elliptical base with a major diameter of 25 feet in the plane of the assumed spin equator and a minor diameter of 17 feet parallel to the assumed spin axis. If it be assumed that the reinforced area does not buckle under this centrifugal force, the following is true (Figure 4):

$$\begin{aligned} BE &= BC - EC = OC - 810 - 26\tan\phi' \\ &= \frac{810}{\cos\phi'} - 810 - 26 \times 0.124 = 3 \text{ in.} \end{aligned} \tag{15}$$

Hence, the maximum deviation of the beacons from the sphere is approximately 3 inches. If the axis of spin forms a relatively large angle with the polar axis of the balloon, the centrifugal force due to the extra reinforcement mass at the polar caps (about 2 pounds) will cause local buckling of somewhat smaller extension and maximum deviation. This maximum deviation is by no means the depth of any wrinkle of the buckled area. Such a distance is estimated to be of the order of a fraction of an inch.

So far it was assumed that the satellite skin is fully rigidized and also that the spin axis passes through the polar caps. By assuming that both the beacons and the pole caps are on the spin equator, two other regions of roughness (at the pole caps), whose extensions are much smaller than the ones at the beacons, could be determined. There is no reason to believe that the spin axis was fixed with respect to the satellite during the spin-up process. Therefore, it is quite feasible that the roughness caused by the centrifugal forces extends over the entire satellite surface.

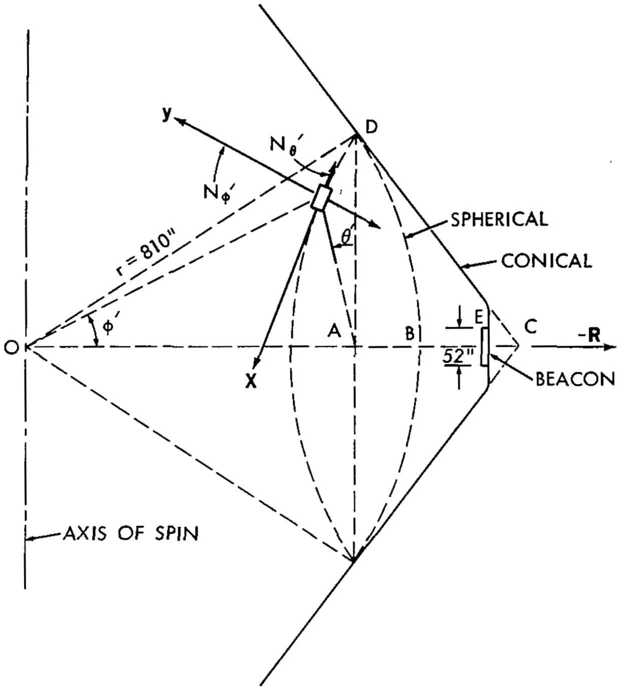

Figure 4–Schematic configuration of spinning satellite.

CONCLUSIONS

It is concluded from the theoretical analysis that the spin of the Echo II satellite could have four distorted areas, one major distortion at each beacon, and one minor distortion at each pole cap. In addition, the centrifugal forces acting on the satellite may have resulted in minor wrinkling of other areas of the satellite surface.

It may be noted that the analysis of the radar data is in agreement with the results obtained in this analysis.

(Manuscript received July 30, 1965)

Appendix A

List of Symbols

C	constant of integration
D	bending stiffness
E	modulus of elasticity
F	centrifugal force
K	extensional stiffness
m	adjusted mass of beacon
N	force per unit length of membrane
$N_\theta, N_\phi, N_{\phi\theta}$	normal and shear membrane forces per unit width; see Figure 3
P_c	calculated buckling pressure
-R	centrifugal force acting on beacon
r	radius of sphere
t	thickness of shell
V	meridional displacement
W	radial displacement
Z	radial body force
ϵ	strain
θ, θ'	angle between meridian through a point and arbitrary reference meridian
μ	Poisson's ratio
ρ	mass per unit area
ϕ, ϕ'	angle between position vector and spin axis
ω	angular velocity

NASA-Langley, 1966

3/22/05

> *"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."*
>
> —NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Technical information generated in connection with a NASA contract or grant and released under NASA auspices.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

TECHNICAL REPRINTS: Information derived from NASA activities and initially published in the form of journal articles.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities but not necessarily reporting the results of individual NASA-programmed scientific efforts. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D.C. 20546