ABSTRACT 35314

A simple expression for the go, no-go criterion is derived in analytical form. This provides a better understanding of the problems involved which is lacking when large computer programs are used. A comparison between "slide rule" results and computer results is indicated on Figure 3.

An example is given using a 185 km near circular parking orbit. It is shown that a 3σ speed error of 6 m/s and a 3σ flight path angle error of 4.5 mrad result in a 3σ perigee error of 30 km which is tolerable when a 5 day orbital life time of the Apollo (SIVB + Service + Command Module) is required.

GO, NO-GO FOR THE APOLLO SPACECRAFT -

AN ANALYTICAL SOLUTION

INTRODUCTION

The purpose of this paper is to derive, in simple form, an analytical expression of the go, no-go criterion for the insertion of the Apollo spacecraft into an earth parking orbit. This means in essence the determination of the maximum perigee height error ($3\sigma h_p$) which can be allowed when a certain orbital lifetime is required after insertion. By doing so, a better understanding of the important physical parameters involved can be obtained, which is normally lacking when large computer programs are used. σ = sigma

Considering only the safety of the astronauts, one can state that the inclination of the parking orbit is of no influence; and only the spacecraft orbital lifetime, which is independent of the inclination, is of importance. The lifetime, on the other hand, is directly related to the perigee height h_p as shown in References 1 and 2.

In brief, if the injection height h is assumed to be 185 km (100 nmi), the minimum perigee height h_p can be determined providing a certain orbital lifetime is assumed (see references 1 and 2). Please note that all the considerations here are made for near circular parking orbits, which are required for the Apollo spacecraft.

[redacted]

This analysis can be accomplished by deriving an expression for the variational equation of the perigee height (equation (13)), and in addition, it can be used to derive a simple error equation as expressed by (14).

I. VARIATIONAL EQUATION FOR THE PERIGEE HEIGHT h_p.

Since the go, no-go decision depends on the perigee height variation Δh_p and ultimately its error σh_p, this value will be derived. Knowing this quantity as a function of the insertion parameters, ρ, v, and γ as shown in Figure 1, the go, no-go criterion can be established; or the errors in ρ, v, and γ, can be determined for a certain error σh_p of the perigee height h_p corresponding to a given or previously established orbital life-time.

Three quantities, insertion radius ρ, insertion speed v, and insertion flight path angle γ determine the ellipse of the parking orbit and thus the perigee radius ρ_p, that is:

$$\rho_p = F(v, \rho, \gamma) \qquad (1)$$

ρ = rho
γ = gamma

No perturbing forces will be considered since only the error in ρ_p (or h_p) is of interest and this error is almost independent of the perturbations.

Varying equation (1) results in:

$$\Delta h_p = \frac{\partial F}{\partial v}\Delta v + \frac{\partial F}{\partial \rho}\Delta h + \frac{\partial F}{\partial \gamma}\Delta \gamma \qquad (2)$$

ρ = rho
γ = gamma

where Δh_p and Δh have replaced the values $\Delta \rho_p$ and $\Delta \rho$ as derived from $\rho = (R + h)$ and $\rho_p = (R + h_p)$.

Starting with the fundamental equations for the ellipse, which can be found in all basic textbooks (see References 1, 3, 4, and 5) on orbits, one can write:

a. The equation for the eccentricity

$$(1 - e^2) = \frac{(v \rho \cos \gamma)^2}{a \mu} \qquad (3)$$

ρ = rho
γ = gamma
μ = mu

b. The equation for the perigee radius

$$\rho_p = a(1-e) \tag{4}$$

ρ = rho

c. The vis viva integral

$$v^2 = \mu\left(\frac{2}{\rho} - \frac{1}{a}\right) \tag{5}$$

μ = mu
ρ = rho

d. The polar equation

$$\rho = \frac{a(1-e^2)}{1 + e\cos\theta} \tag{6}$$

ρ = rho
θ = theta

where $\mu = 3.987 \times 10^5 \, km^3/sec^2$ is the gravitational parameter of the earth (see References 1 and 6), and θ is the true anomaly of the elliptic orbit under consideration.

μ = mu
θ = theta

Equation (1) can be obtained in a straightforward manner using equations 3, 4, and 5, that is:

$$2\rho_p - 2\frac{\rho_p^2}{\rho} + \frac{\rho_p^2 v^2}{\mu} = \frac{1}{\mu} v^2 \rho^2 \cos^2\gamma \tag{7}$$

ρ = rho
μ = mu
γ = gamma

In order to obtain a variational expression for ρ_p or h_p, equation (7) has to be varied, resulting in:

$$\Delta h \left[1 - 2\frac{\rho_p}{\rho} + \frac{v^2 \rho_p}{\mu}\right] = \left[\frac{(v\rho \cos\gamma)^2}{\mu} - \frac{\rho_p^{\,2}}{\rho}\right]\frac{\Delta h}{\rho}$$

$$+ \left[\frac{(v\rho \cos\gamma)^2}{\mu} - \frac{v^2 \rho_p^{\,2}}{\mu}\right]\frac{\Delta v}{v} - \left[\frac{(v\rho \cos\gamma)^2}{\mu} \tan\gamma\right]\Delta\gamma \tag{8}$$

ρ = rho
μ = mu
γ = gamma

where proper terms have been collected as shown in equation (8). Please note that $\Delta\rho = \Delta h$ and $\Delta h_p = \Delta\rho_p$ because their relationship is linear. In the following, it is shown how the coefficients in equation (8) can be evaluated. In order to simplify these, it is assumed that the orbit eccentricities are within $0 \le e \le 0.15$, which includes all of the ranges of the parking orbits under consideration for the Apollo spacecraft.

Considering only terms up to the second order in e, that is $e^2 \neq 0$ (all higher terms of e are considered small compared to other terms involved), one obtains the following results for the coefficients of equation (8):

First, for Δh:

$$\left(1 - 2\,\frac{\rho_p}{\rho} + \frac{V^2 \rho_p}{\mu}\right) \doteq e \qquad (9)$$

ρ = rho
μ = mu

using equation (6) with the assumption that θ is small making $\cos\theta \doteq 1$. This is justified since the normal insertion conditions are $\gamma \doteq 0$ or $\theta \doteq 0$ which means that $\rho_p \doteq \rho$ and $\frac{V^2 \rho_p}{\mu} \doteq (1 + e)$

ρ = rho
μ = mu

Second, for $\left(\frac{\Delta \rho}{\rho}\right) = \left(\frac{\Delta h}{\rho}\right)$:

$$\left[\frac{(V\rho\cos\gamma)^2}{\mu} - \frac{\rho_p^2}{\rho}\right] = e\rho \qquad (10)$$

ρ = rho
γ = gamma
μ = mu

using equation (1), introducing $\frac{\rho_p^2}{\rho} \doteq (1 + e)$

ρ = rho

and neglecting terms of e^3 and higher.

Third, for $\left(\frac{\Delta V}{V}\right)$:

$$\left[\frac{(V\rho\cos\gamma)^2}{\mu} - \frac{V^2 \rho_p^2}{\mu}\right] = 2ea = 2e\rho(1+e) \qquad (11)$$

ρ = rho
γ = gamma
μ = mu

using (1), (4), and (5) and setting $(1 - \frac{a}{\rho}) \doteq e$.

Fourth, for $\Delta\gamma$:

$$\left[\frac{(v\rho \cos\gamma)^2}{\mu} \tan\gamma\right] = a(1-e^2) \tan\gamma \qquad (12)$$

ρ = rho
γ = gamma
μ = mu

These coefficients, represented by the equations (9), (10), (11), and (12), can now be introduced into equation (8), which leads to the final variational equation for the perigee height, that is

$$\Delta h_p = \Delta h + 2(1+e)(R+h)\left(\frac{\Delta v}{v}\right) - (1+e)(R+h)\Delta\gamma \qquad (13)$$

γ = gamma

where the value $\left(\frac{\tan\gamma}{e}\right) \doteq 1$ (limit case for $e \to o$ and $\gamma \to 0$) and $\rho = (R + h)$ were introduced.

Equation (13) can be used to determine the necessary "maneuvers" to change the perigee height h_p, by a certain amount Δh_p. by changing Δh, Δv or $\Delta\gamma$ as deemed necessary. This is graphically illustrated in Figure 2. It can be seen from this graph that inadequate insertion speed (Δv negative) can be compensated for by a negative $\Delta\gamma$, etc. For example, using Δh = 0.5 km, a flight path angle variation of -1.5mrad and a velocity variation of -6m/s results in a zero variation of the perigee height, as indicated in Figure 2. Also, this can be seen by inspecting equation (13).

II. THE ERROR IN THE ORBITAL PERIGEE HEIGHT σh_p.

Assuming zero covariances, the error in perigee height σh_p can be written as the sum of the squares of the variational expressions, (12), that is

$$\sigma h_p = (R+h)\left[\left(\frac{\sigma h}{R+h}\right)^2 + 4(1+e)^2\left(\frac{\sigma v}{V}\right)^2 + (1+e)^2\sigma\gamma^2\right]^{1/2} \quad (14)$$

This equation is presented in graph form in Figure 3, for a typical Apollo parking orbit, $e \doteq 0$, $h = 185$ km. It is shown in References 7 and 8, that the neglection of the covariances always gives conservative estimate of the errors volume.

The corresponding values obtained with a computer program in conjunction with a Monte Carlo approach are shown in Figure 3. This substantiates what was mentioned in the previous paragraph.

III. CRITERION FOR GO, NO-GO IN SIMPLE FORM.

The Apollo spacecraft has a weight of approximately 91,000 kg at the time of insertion into the earth parking orbit, and a frontal area of about 35 m^2.

Using data as given in References 1 and 2, one obtains a lifetime of approximately 30 days for a 185 km (100 nmi) orbit, which corresponds to the area previously mentioned. For a 160 km (88 nmi) orbit, the lifetime is approximately 9 days (respectively). See Figures 4 and 4a.

As an example, assume that for a go, no-go decision a 5-day orbital lifetime is required. Using a 185 km (100 nmi) circular orbit as the nominal parking orbit, one could tolerate a $3\sigma h_p$ error of approximately 30 km from the lifetime considerations. (See Figure 4.) Using Figure 3, one finds that a 3σ speed error up to 6 m/s and a 3σ flight path angle error up to 4.5 mrad are tolerable; or a 3σ speed error of 12 m/s and a 3σ flight path angle error of 3 mrad are also within the $3\sigma h_p$ error bounds. In all the cases, the perigee height error of $3\sigma h_p$ was assumed to be 1.5 km. The value for the insertion height can be easily obtained as shown in Reference 9 ($1\sigma h \doteq 500$ m). Inspection of Figure 3 shows that the error in insertion height does not play an important role; $3\sigma_h = 1.5$ km or $3\sigma_h = 3$ km does not alter the conclusions reached. In Reference 9 the velocity errors $1\sigma v$ are presented for different noise, bias and station locations (ships) using one tracking sample per second without a priori knowledge of the orbit.

Using the 3σ error analysis results just mentioned, for speed and flight path angles given in Reference 10, for a 40 second tracking interval and 5 tracking samples per second, that is 4 m/s and 2 mrad, shows that the associated 3σ error in perigee height is well within the assumed limits. Therefore, a safe go, no-go can be made using a tracking time of approximately 40 seconds with 5 tracking samples per second. The values stated are based upon radar measurements with a range noise error of $1\sigma r = 10$ m and angular noise errors of $1\sigma\alpha = 1\sigma\epsilon = 0.4$ mrad combined with bias errors twice the stated noise values respectively. A detailed table on the tracking errors used for Apollo is presented in Reference 11, the Apollo Navigation Working Group Report.

α = alpha
ϵ = epsilon

It is interesting to note that the location errors for the tracker (a ship in case of Apollo, see References 9 and 10) do <u>not</u> influence the velocity and flight path angle, which are the vital parameters for this decision. As clearly stated in Table 3.4 of Reference 10, speed, flight path angle and altitude errors are not affected much by the station location error. A numerical example is given for better understanding. Assume the insertion ship tracks the spacecraft with its C-band radar using the errors quoted before and in addition has a navigational error of 3_σ = 4 km (in N and E direction). The corresponding error in spacecraft speed $3_{\sigma_V} \doteq 2$ m/s and that in flight path angle $\sigma\gamma \doteq 1$ mrad after 2 minutes of tracking. This can also be seen from Figures 5 and 6 in Reference 9, which show that the navigation accuracies needed for the Apollo insertion ship (References 9 and 10) are only moderate.

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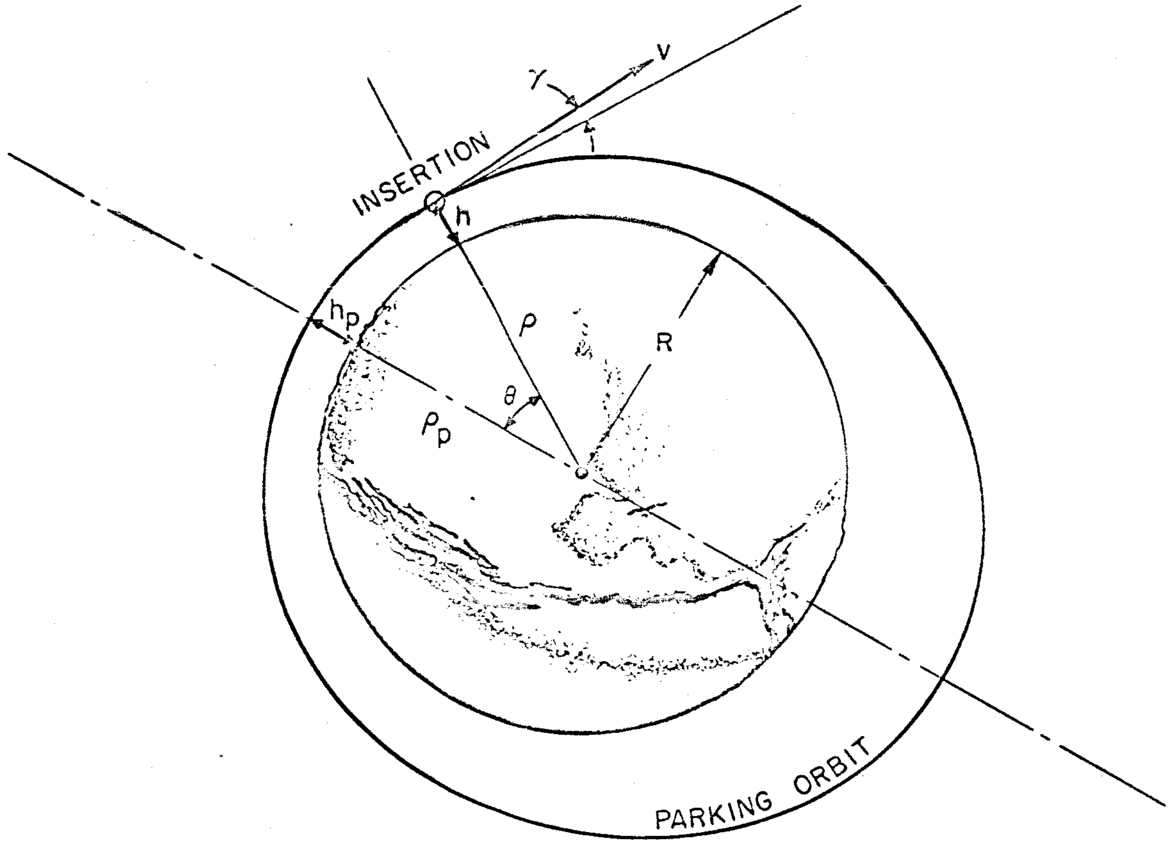

h_p --- PERIGEE HEIGHT

h --- INSERTION HEIGHT

v --- INSERTION SPEED

γ --- INSERTION FLIGHT PATH ANGLE

R --- EARTH RADIUS

ρ --- (R+h)

ρ_p --- $(R+h_p)$

θ --- TRUE ANAMOLI

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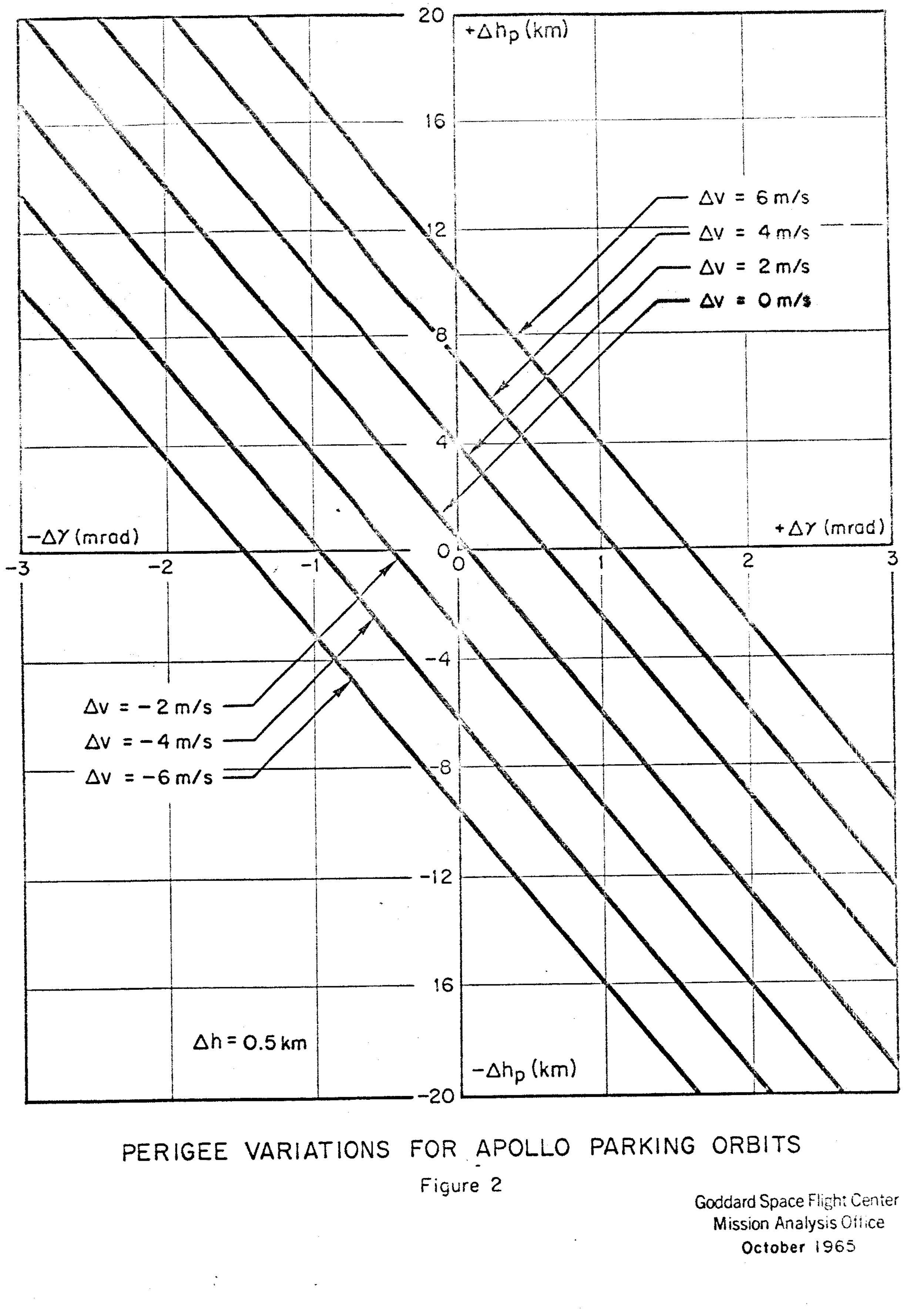
ORBITAL INSERTION GEOMETRY

Figure 1



PERIGEE VARIATIONS FOR APOLLO PARKING ORBITS

Figure 2

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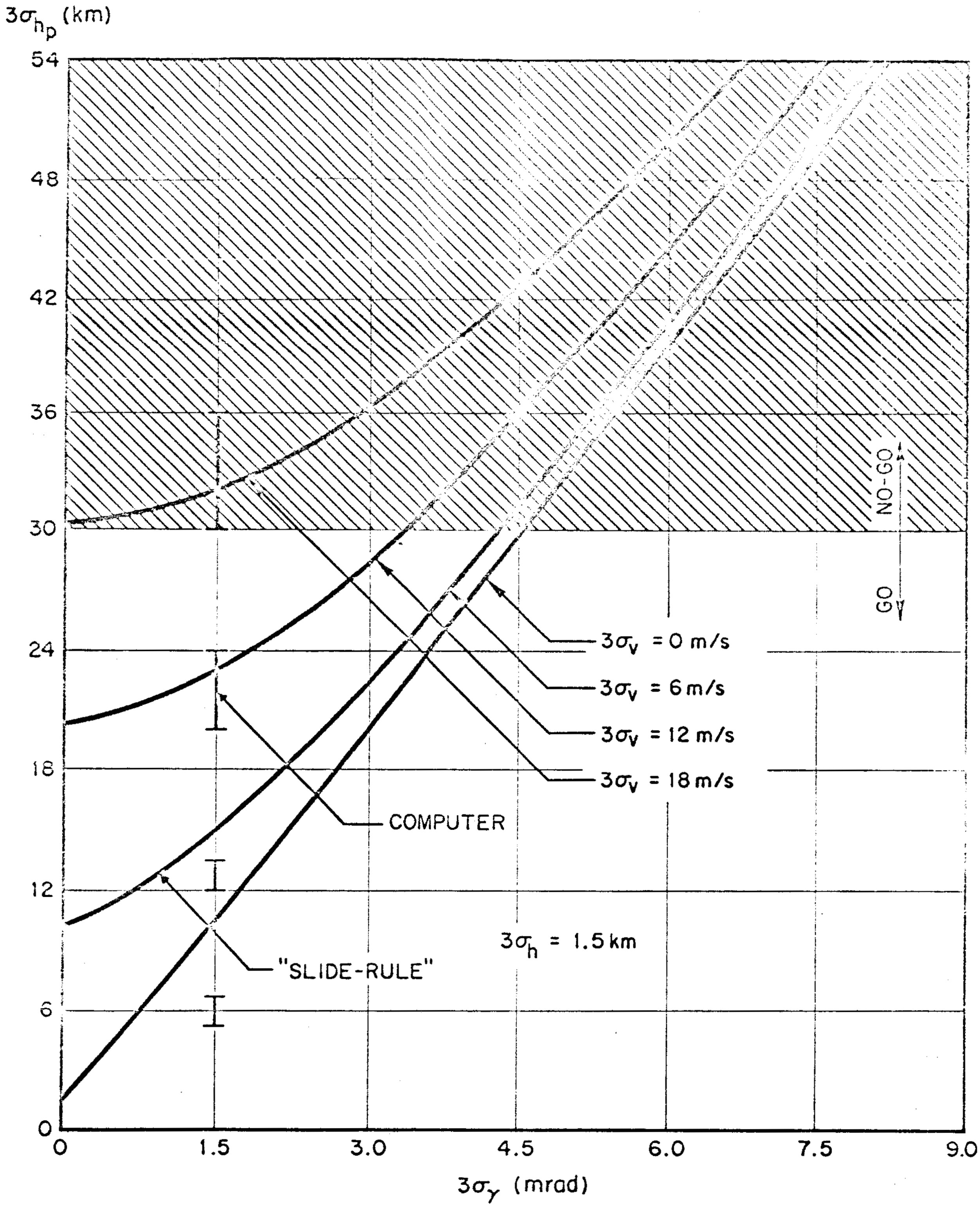

PERIGEE ERROR FOR APOLLO PARKING ORBITS

($e \doteq 0$, $h_o = 185$ km)

Figure 3

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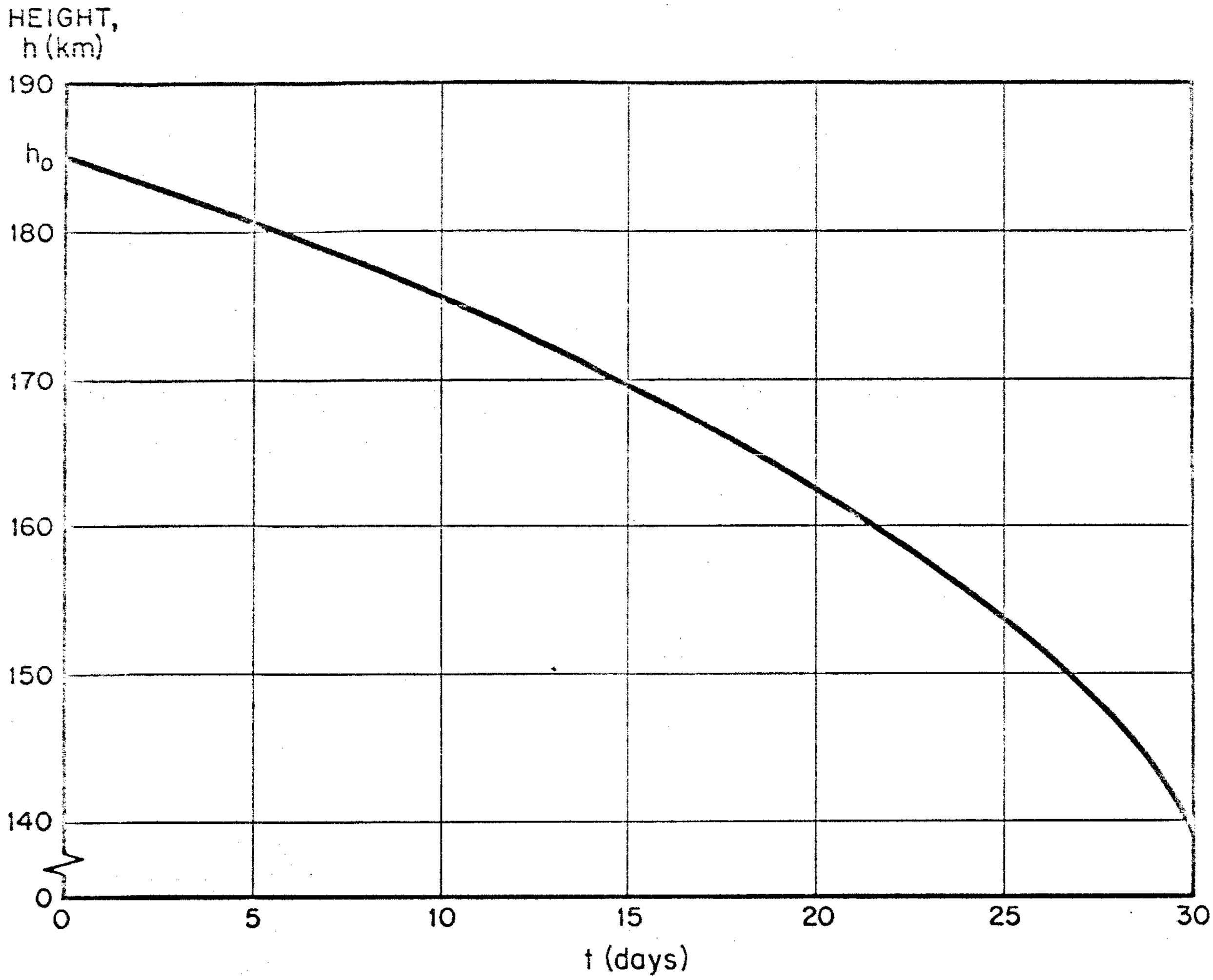

INITIAL HEIGHT, $h_0 = 185$ km

INITIAL ECCENTRICITY, $e_0 \doteq 0$

INITIAL AIR DENSITY, $\rho_0 = 2.42 \times 10^{-10}$ kg/m^3

FRONTAL AREA, $A_1 = 35$ m^2

WEIGHT IN ORBIT, W = 91,000 kg

DRAG COEFFICIENT, $C_D = 2$.

$$\frac{C_D A_1}{W} = 0.769 \times 10^{-3} \text{ m}^2/\text{kg}$$

HEIGHT OF THE APOLLO PARKING ORBIT AFTER INSERTION
(Perkins, Ref. 2)

Figure 4

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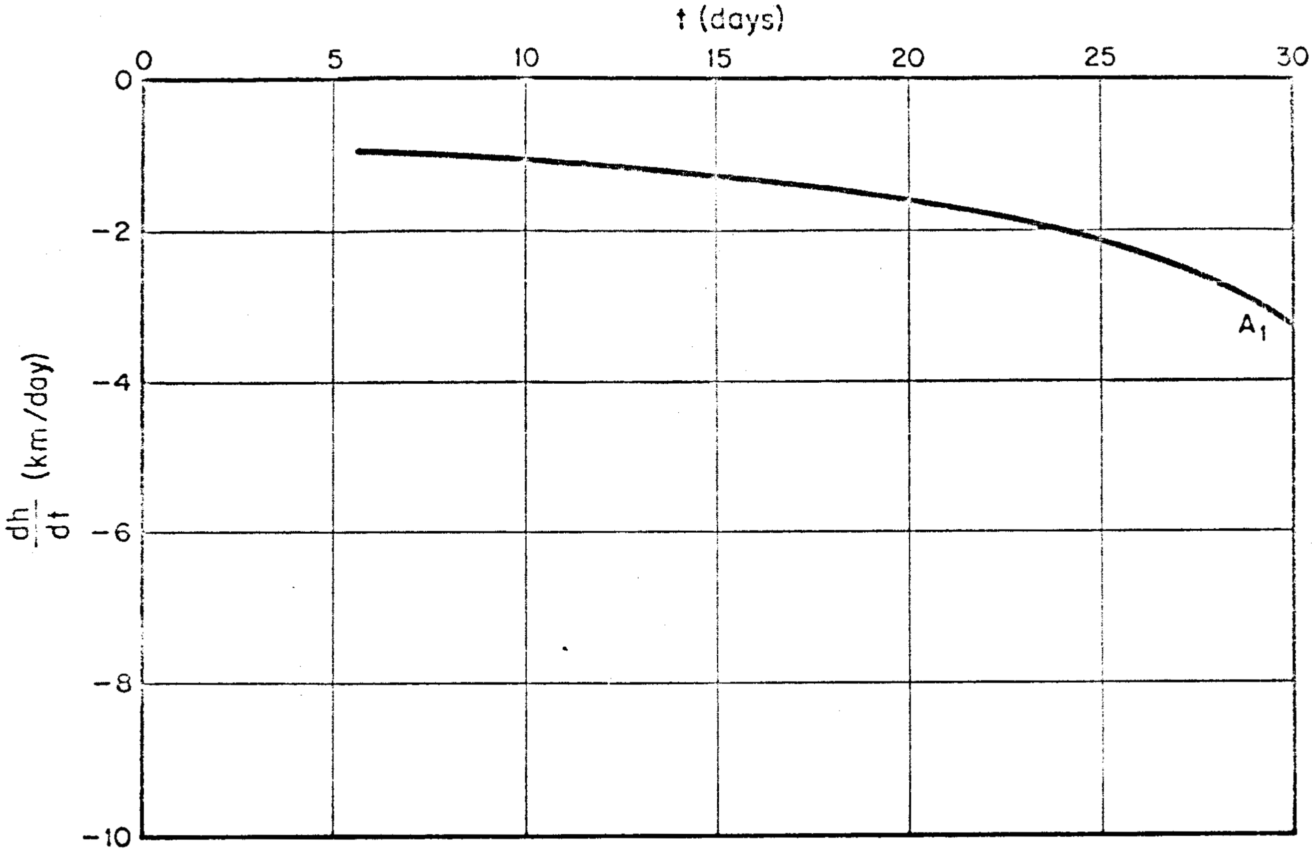

INITIAL HEIGHT, $h_0 = 185$ km

INITIAL ECCENTRICITY, $e_0 \doteq 0$

INITIAL AIR DENSITY, $\rho_0 = 2.42 \times 10^{-10}$ kg/m^3

FRONTAL AREA, $A_1 = 35$ m^2

WEIGHT IN ORBIT, W = 91,000 kg

DRAG COEFFICIENT, $C_D = 2$

$$\frac{C_D A_1}{W} = 0.769 \times 10^{-3} \text{ m}^2/\text{kg}$$

HEIGHT RATES FOR THE APOLLO PARKING ORBIT
(Perkins, Ref. 2)
Figure 4a

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