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Subject of Report	The Surface Roughness of Echo II- A Preliminary Study
Submitted by	R. T. Compton, Jr. Antenna Laboratory Department of Electrical Engineering
Date	31 December 1964

ABSTRACT

This report deals with the surface roughness of Echo II. It is shown that when a cw signal is reflected from Echo II, the power spectral density of the returned signal is related to the scattering cross-section per unit area of the Echo surface by an Abel Integral equation. The solution to the integral equation is used to compute the scattering function from experimentally obtained curves of power spectral density. The results indicate that the RMS slope of the surface of Echo II is approximately 1.5°.

TABLE OF CONTENTS

	Page
INTRODUCTION	1
PRESENTATION OF THEORY	2
EXPERIMENTAL RESULTS	9
CONCLUSIONS	12
REFERENCES	14

THE SURFACE ROUGHNESS OF ECHO II - A PRELIMINARY STUDY

INTRODUCTION

Communication experiments presently being conducted on Echo II have two purposes: to evaluate the characteristics of Echo II as a communication channel and to determine the physical condition of the balloon itself. Of course, these two areas of research overlap somewhat, since the communication capability is related to the surface condition of the balloon.

This report deals with the surface roughness of Echo II. When an unmodulated carrier is transmitted over the Echo link, considerable amplitude scintillation is observed in the return signals[1]. This scintillation is believed to be due to the fact that the surface of the balloon has slight roughness and the balloon is rotating (in addition to it orbital velocity), thus causing a frequency dispersion in the reflected signal. Although it is not known what caused the balloon to acquire a spin, it has been determined from telemetry data that Echo II rotates with a period of approximately 100 seconds[2]. Also, Julian and Hynek [3] have noticed a periodicity in the signal fluctuations of 104 seconds. Knowing the spin rate of the balloon makes it possible to calculate (after certain assumptions are made) the scattering cross-section per unit area for the surface, which can then be used to estimate surface roughness. Unfortunately in the case of Echo I the spin rate is unknown, (at least to this author) which makes it more difficult to estimate surface roughness.

In this report, the scattering cross-section function is defined and the relation between it and the power density spectrum of the received signal is derived, for the case of a monostatic radar. The result is in the form of an Abel integral equation, which is easily solved. By this method the scattering cross-section function of Echo II can be found from the power density spectrum of the received signal. The scattering function may then be used to estimate the surface roughness of the balloon.

Experimental curves of Power Density Spectra for revolution numbers 1730 and 1836 are presented and are used to calculate the scattering function. The data obtained during these two passes were typical of that taken during most passes.

PRESENTATION OF THEORY

When a radio signal is reflected from Echo II, two types of doppler shift occur. First, there is a large overall doppler shift due to the orbital velocity of the satellite (50 kc is typical at 2 kmc) and second, there is a frequency dispersion (or smearing of the spectrum) due to the rotation of the balloon about its spin axis and the roughness of the balloon surface. For the case of an unmodulated transmitted signal, the power density spectra of the incident and reflected signals are shown qualitatively in Fig. 1. In this report, only the spectral

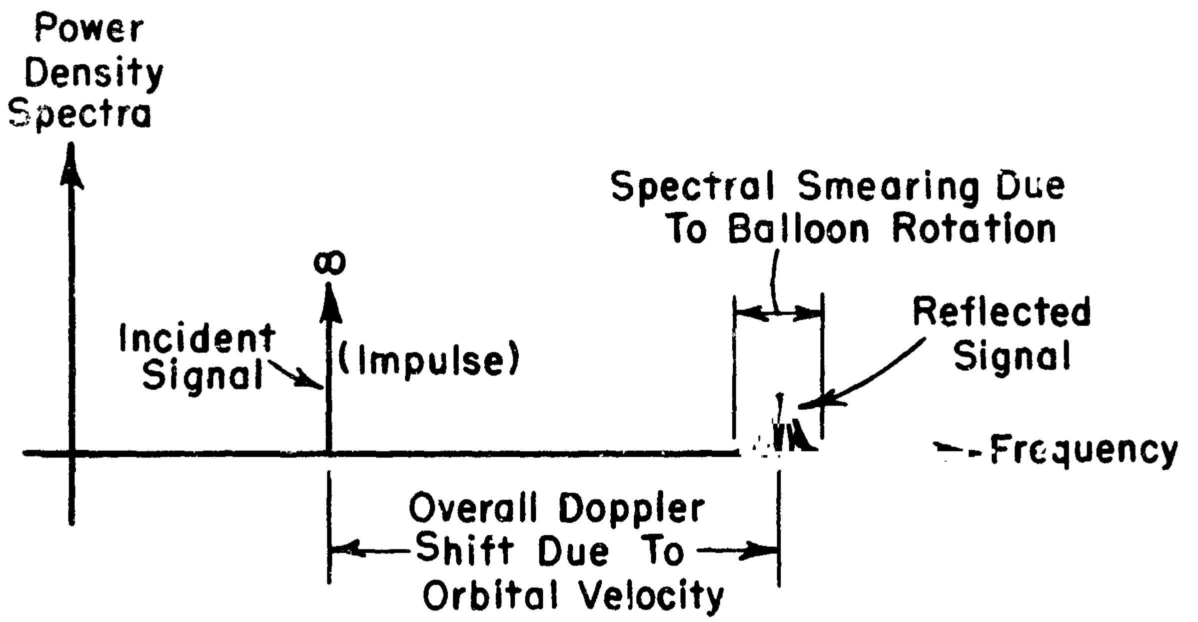

Fig. 1. Power density spectra of incident and reflected signals.

smearing due to rotation will be considered. At The Ohio State University Satellite Communication Center, the overall doppler shift due to orbital velocity is removed by a tracking oscillator, and hence does not appear at the receiver output.

The time-average signal power received from Echo II is related to the time-average scattering cross-section of the satellite, $\bar{\sigma}_t$, through the well-known radar range equation. By calibrating the system parameters in a given experiment, the absolute value of $\bar{\sigma}_t$ may be computed. This calculation is the subject of other reports[5,6,7] and will not be treated here.

Let an xyz coordinate system be placed at the center of the spherical satellite, with the x-axis chosen to lie along the direction of incidence. We define the function $\gamma(y, z)$ as the average backscatter cross-section per unit projected area. That is, $\gamma(y, z)dydz$ represents the contribution to the total average cross-section $\bar{\sigma}_t$ from the portion of the projected surface area lying between y and y + dy and between z and z + dz. This situation is shown in Fig. 2

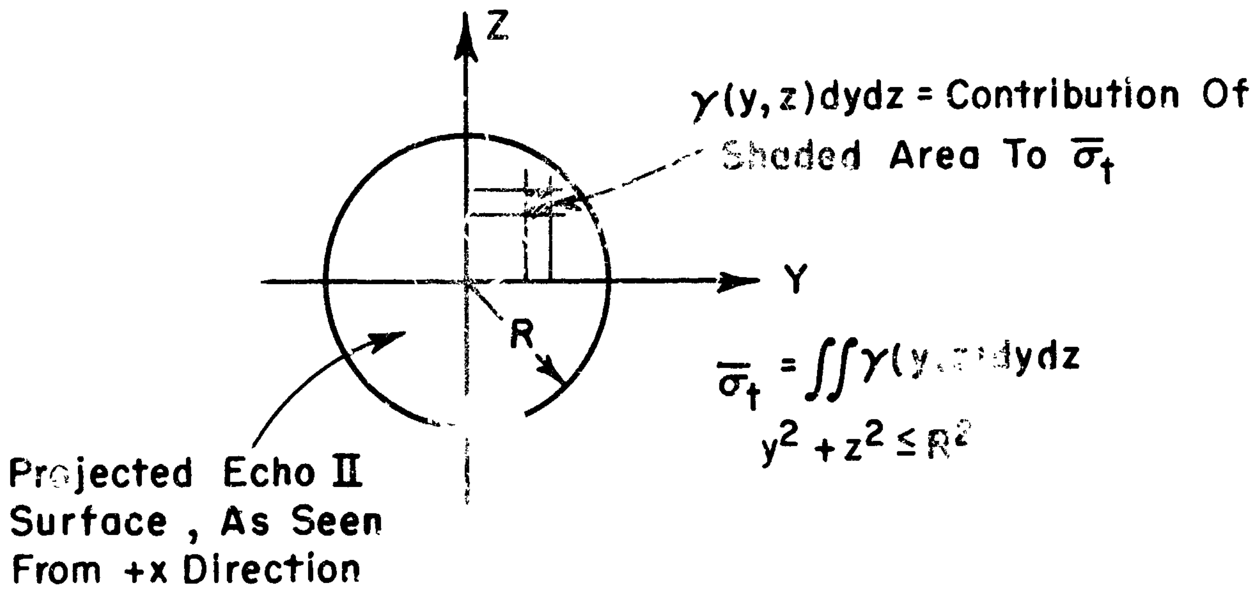

Fig. 2. Projected area of Echo II surface.

Without loss of generality, the angular velocity vector due to the satellite spin, $\underline{\Omega}$, may be supposed to lie in the xz-plane, making an angle τ with the z-axis, as shown in Fig. 3.

Then the x-component of velocity (the component causing doppler shift) of a point on the surface of the sphere is given by

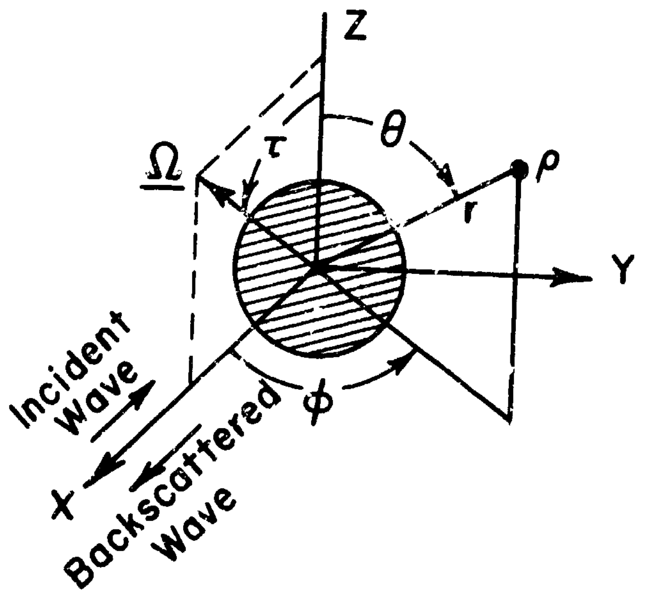

Fig. 3. The angular velocity $\underline{\Omega}$.

(1) $v_x = -\Omega y \cos \tau$

where

v_x = the x-component of velocity and $\Omega = |\underline{\Omega}|$. The doppler shift of the signal reflected from a point (y, z) is then

(2) $$\Delta\omega = \frac{2\omega_o}{c} v_x = -\frac{2\omega_o \Omega}{c} y \cos \tau = Ky$$

where

$\Delta\omega$ = doppler shift (rad./sec.)

ω_o = carrier frequency (rad./sec.)

c = velocity of light

and

$$K = -\frac{2\omega_o\Omega}{c}\cos\tau .$$

Thus all points on the surface of the satellite with a given y-coordinate cause the same doppler shift.

Let $P(\omega)$ be the one-sided power density spectrum of the received signal; i.e.,

(3) $$\text{Average Received Power} = \int_0^\infty P(\omega)d\omega .$$

Define the function F(y) as

(4) $$F(y) = P(\omega)\Big|_{\omega=\omega_o + Ky}$$

(5) $$= P\left[\omega_o\left(1 - \frac{2\Omega}{c}y\cos\tau\right)\right] .$$

We will call F(y) the <u>doppler function</u>. Aside from a constant due to the parameters in the radar range equation, F(y)dy is the contribution to the total cross-section of all points on the satellite surface lying between y and y + dy, i.e. all points with a given doppler shift. Thus

(6) $$F(y) = \int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \gamma(y,z)dz$$

where R is the radius of the sphere.

Finally, we make two assumptions. First, we assume that the value of $\gamma(y,z)$ actually depends only on the angle "α" between the unit vector $\hat{x}$ and the normal to the spherical surface, $\hat{r}$, as shown in Fig. 4. Then the value of $\gamma(y,z)$ may be denoted by $\gamma_o(\alpha)$, a function only of α. And secondly, we make an assumption which is actually implicit in the above assumption, namely that the function $\gamma_o(\alpha)$ is the <u>same function</u> for all points of the surface. For a very rough object, such as the moon, these conditions would not hold, but it is felt that in the case of Echo II, which is much smoother, they will be realistic. With these assumptions,

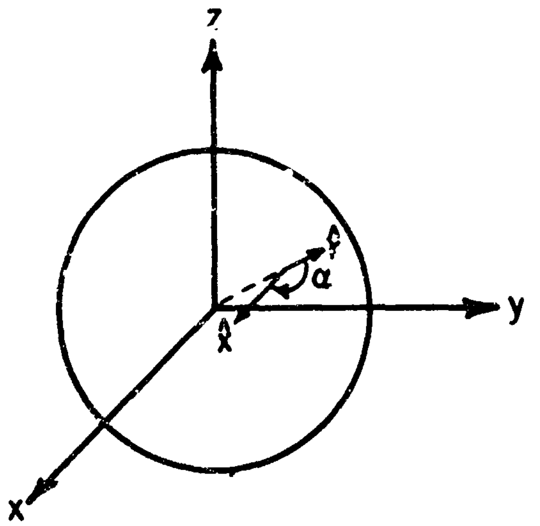

Fig. 4. The angle "α".

Eq. (6) may be rewritten as follows. With (r, θ, ϕ) as defined in Fig. 3,

(7) $$\hat{r} = \hat{x} \sin\theta \cos\phi + \hat{y} \sin\theta \sin\phi + \hat{z} \cos\theta ,$$

and for any point on the surface of the sphere,

(8) $$\cos\alpha = \hat{x} \cdot \hat{r} = \sin\theta \cos\phi$$

or

(9) $$\cos^2\alpha = \sin^2\theta \cos^2\phi .$$

Also, for a point (on the sphere) with coordinate y,

(10) $$\left(\frac{y}{R}\right)^2 = \sin^2\theta \sin^2\phi .$$

Combining Eqs. (9) and (10) gives the relation

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left[\cos^2\alpha + \left(\frac{y}{R}\right)^2\right] = \sin^2\alpha - \left(\frac{y}{R}\right)^2 \tag{11}$$

so that the z-coordinate of the point is given in terms of y and α as

$$z = R\cos\theta = \sqrt{R^2\sin^2\alpha - y^2} \ . \tag{12}$$

Hence also

$$dz = \frac{R^2 \sin\alpha \cos\alpha \, d\alpha}{\sqrt{R^2\sin^2\alpha - y^2}} \tag{13}$$

and Eq. (6) may be written

$$F(y) = 2 \int_{\alpha=\sin^{-1}\left(\frac{y}{R}\right)}^{\pi/2} \gamma_o(\alpha) \frac{R^2\sin\alpha\cos\alpha}{\sqrt{R^2\sin^2\alpha - y^2}} \, d\alpha \ . \tag{14}$$

Since F(y) is known (i.e., measured), this is an integral equation for $\gamma_o(\alpha)$. By inverting Eq. (14), $\gamma_o(\alpha)$ may be calculated from F(y).

Equation (14) may be solved as follows. Making the change of variables

$$y = R\sqrt{\xi} \tag{15}$$

$$\sin^2\alpha = \eta \tag{16}$$

transforms Eq. (14) into

$$F\left(R\sqrt{\xi}\right) = R\int_{\eta=\xi}^{\eta=1} \gamma_o\left(\sin^{-1}\sqrt{\eta}\right) \frac{1}{\sqrt{\eta-\epsilon}} \, d\eta \ . \tag{17}$$

Next, defining the functions

$$(18) \qquad g(\eta) = \gamma_0(\sin^{-1}\sqrt{\eta})$$

$$(19) \qquad f(\xi) = \frac{F(R\sqrt{\xi})}{R}$$

gives

$$(20) \qquad f(\xi) = \int_{\eta=\xi}^{\eta=1} g(\eta) \frac{1}{\sqrt{\eta-\xi}} \, d\eta$$

which is recognized as a form of Abel's Equation. The solution is easily obtained [8, 9]; it is

$$(21) \qquad g(\eta) = -\frac{1}{\pi} \frac{d}{d\eta} \int_{\xi=\eta}^{1} \frac{f(\xi)}{\sqrt{\xi-\eta}} \, d\xi .$$

Finally, using the inverse relations

$$(22) \qquad \alpha = \sin^{-1}\sqrt{\eta}$$

$$(23) \qquad \xi = \frac{y^2}{R^2}$$

gives

$$(24) \qquad \gamma_0(\alpha) = -\frac{2}{\pi R^2} \left[\frac{d}{d\eta} \int_{y=R\sqrt{\eta}}^{R} \frac{y\, F(y)dy}{\sqrt{y^2 - R^2 \beta}} \right]_{\eta=\sin^2 \alpha}$$

from which the scattering function $\gamma_0(\alpha)$ may be calculated from the power density spectrum. This calculation is done in the next section for two typical experimental power density spectra curves taken on Echo II.

EXPERIMENTAL RESULTS

In Figs. 5 and 6 are shown two typical curves of power spectral density versus frequency. These experimental curves were obtained during Revolution numbers 1730 and 1836 of Echo II. The satellite was illuminated with an unmodulated carrier at 2270 mc/sec from a transmitter at Ohio University, Athens, Ohio. The reflected signal was received at Columbus, Ohio at The Ohio State University Satellite Communication Center[10]. The signal was processed through an envelope detector and recorded continuously on magnetic tape for later analysis.

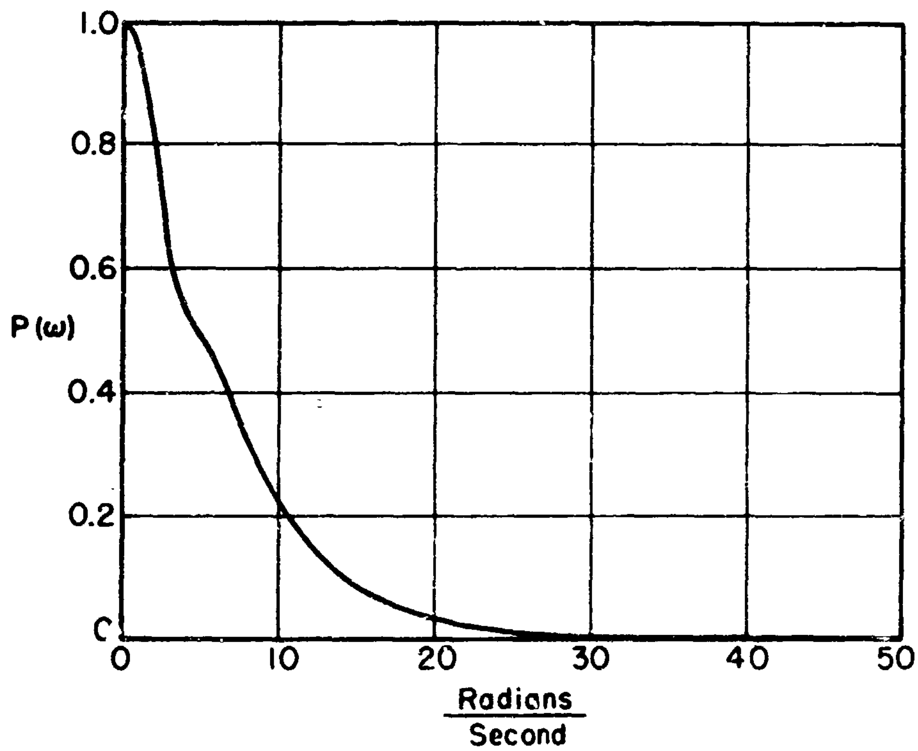

Fig. 5. Relative power spectral density of Echo II signals (Revolution No. 1730).

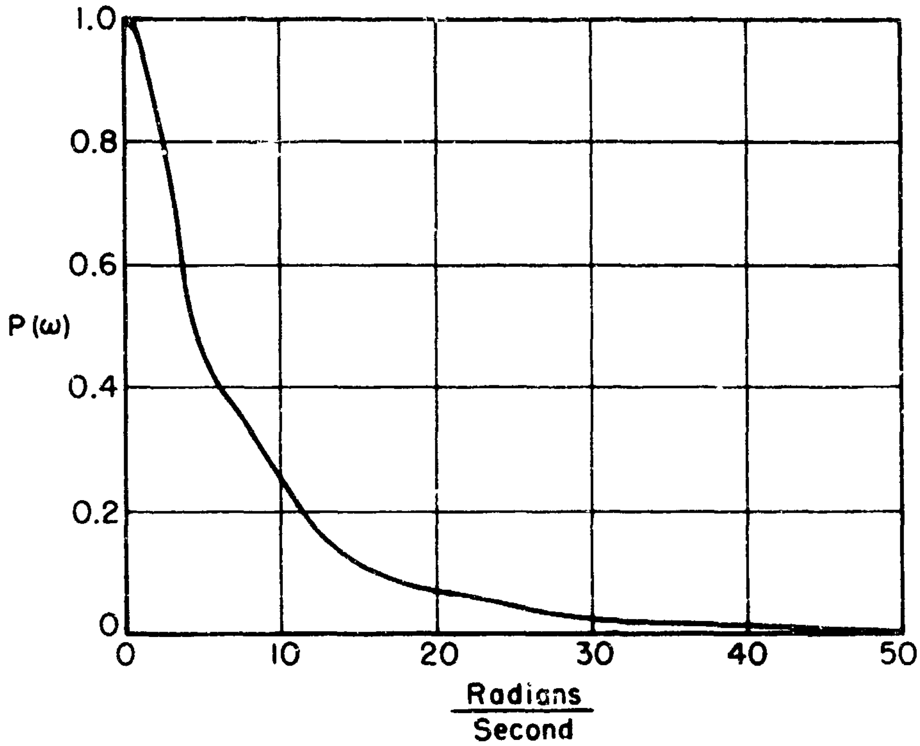

Fig. 6. Relative power spectral density of Echo II signals (revolution No. 1836).

The recorded signal strength fluctuations were sampled at a rate of 50 samples per second and processed on the Ohio State University IBM 7094 digital computer to obtain the autocorrelation function of the signal[11,12,13,14,15]. The power spectral density curves were then computed as the Fourier Transforms of the autocorrelation functions.

The data from revolution numbers 1730 and 1836 have been selected (more or less arbitrarily) as being representative of similar data obtained for many passes[16,17]. The above description of the data-processing techniques used is not intended to be complete but only to serve as a brief general discussion. More detailed descriptions of the data analysis may be found in several previous reports[11-17].

The results of applying Eq. (24) to the curves of Figs. 5 and 6 are shown in Figs. 7 and 8. It is seen that for angles of incidence α greater than 10° (from the normal), there is no significant backscatter from the rough surface. The scattering function has dropped to a value of 0.5 by 1.5°. Considering the surface of the balloon to have a height which varies randomly about a true spherical shape, we may use the angle α at which

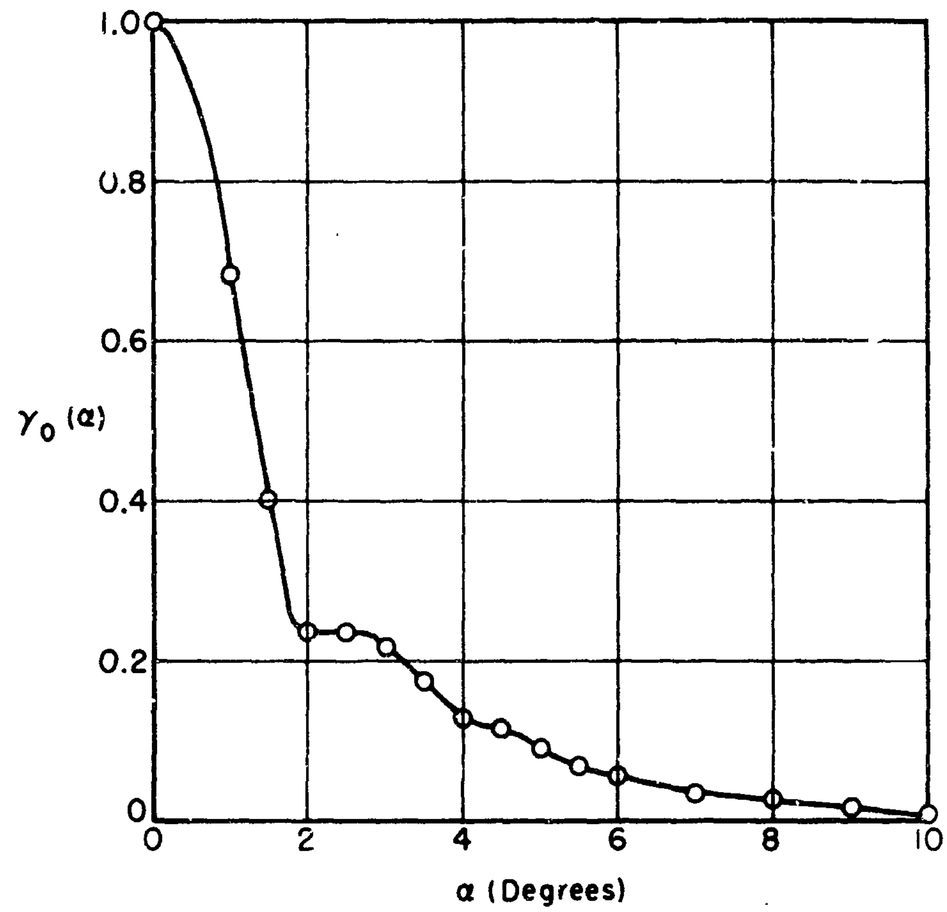

Fig. 7. The scattering function $\gamma_0(\alpha)$ vs. angle of incidence α for revolution No. 1730.

$\gamma_0(\alpha) = 1/2$ as a rough estimate of the RMS slope of the surface (the slop relative to a perfectly spherical surface). Based on this "rule-of-thumb", the RMS slope of the surface is found to be approximately $1\frac{1}{2}^{o}$. (This rule-of-thumb is based on the experimental results obtained for many types of rough surface scattering measurements made at the Antenna Laboratory. Of course, there is nothing rigorous about this, but it is believed to be an adequate engineering guide.).

The experimental curves obtained in Figs. 7 and 8 can also be compared with theoretical curves which have been obtained by Ott[18]

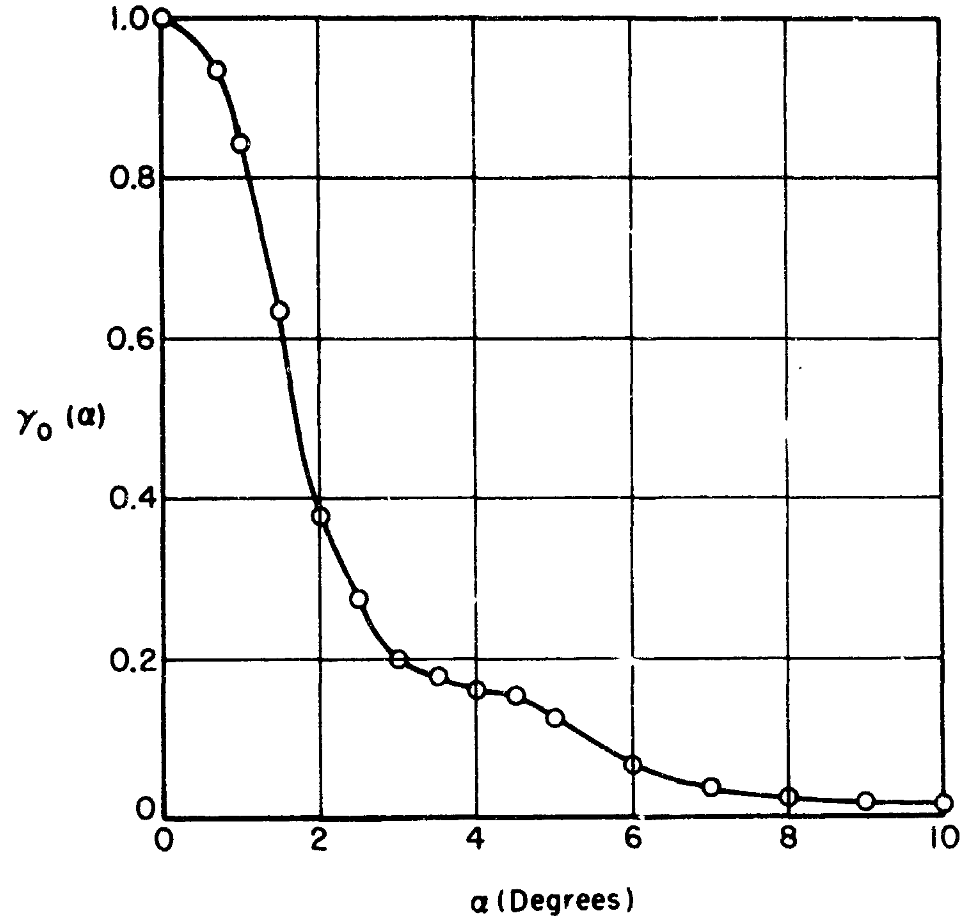

Fig. 8. The scattering function $\gamma_0(\alpha)$ vs. angle of incidence α for revolution No. 1836.

for a number of different mathematical models of a rough surface. Ott's formulas can be made to fit the experimental curves approximately by adjusting the parameters in his formulas. However, since it is not known which mathematical model is the most accurate description of the surface statistics, it is not known whether this method is any more accurate than the above "rule-of-thumb".

CONCLUSIONS

The relationship between the power density spectrum of the echo from a rough rotating sphere and the surface scattering function of the sphere has been derived. This relationship has been shown to be in the form of an Abel integral equation, the solution of which allows the scattering function to be calculated from the power density spectrum.

Experimental data for the power density spectra for two representative passes of Echo II have been used to calculate the scattering function.

The results of this calculation show that there is no significant backscatter for angles of incidence greater than 10° off the normal. The RMS slope of the surface roughness is estimated to be 1.5°.

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